Analog optical computing primitives in silicon photonics

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Optical computing accelerators help alleviate bandwidth and power consumption bottlenecks in electronics. We show an approach to implementing logarithmic-type analog co-processors in silicon photonics and use it to perform the exponentiation operation and the recovery of a signal in the presence of multiplicative distortion. The function is realized by exploiting nonlinear-absorption-enhanced Raman amplification saturation in a silicon waveguide. © 2016 Optical Society of America

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With the proliferation of big data and the rapid increase in power dissipation of electronics, there is renewed interest in the use of optics for computing. In contrast to the optical computing efforts of the past, an all-optical computer may not be the most prudent goal [1–3]. Instead, a hybrid approach where optical systems are selectively applied to alleviate bottlenecks and assist electronic processors is a more fruitful pursuit. The idea of optical co-processors is proposed as hardware accelerators to take part of the processing burden off of the electronic processors [4–6], as shown in Fig. 1(a). Composed of carefully designed photonics components, the optical co-processor performs a certain analog computational operation in real time on the input optical signal before it is acquired and digitized.

Among the analog computing primitives, the logarithmic function is of importance and is one of the most challenging operations to perform in optics. The logarithmic primitive has broad applications, including the log-likelihood estimation for machine learning, recovery of signal with multiplicative distortion, and the exponentiation operation (raising a variable to a given power). As illustrated in Fig. 1(b), an optical implementation of the exponentiation operation can be achieved by three sequential components: the logarithmic primitive, the scaling primitive, and the natural exponentiation primitive. Apart from the logarithmic primitive, the two remaining primitives can be emulated using commercially available optical systems. For example, scaling can be achieved with variable optical attenuators or four-wave mixing, while a Raman amplifier operating in the low depletion regime provides the natural exponentiation function with respect to the input intensity.

The lack of logarithmic dependence in conventional optical interactions renders the realization of a logarithm computation block formidable. Logarithmic filtering was demonstrated in literature using nonlinear photographic films [7] and hologram masks [8], but the cumbersome free space setup and the complicated processing have limited its range of applicability.

In this Letter, we show an approach to approximate the optical input–output relationship as a logarithmic function in a silicon waveguide via numerical studies. Silicon naturally exhibits two-photon absorption (TPA) at telecommunication wavelengths. This nonlinear absorption, which limits the signal's output intensity and is normally deleterious [9–11], becomes a fortuitous natural candidate to approximate the logarithm. In the presence of a suitably wavelength-shifted pump source, stimulated Raman scattering amplifies the signal, and it has such a strong effect that it caused this indirect bandgap material to lase for the first time [12]. When the signal grows strong enough to deplete the pump source, the gain saturates, leading to a second method to achieve logarithm-like behavior. Non-degenerate TPA is also introduced to the system to shape the output curve. Using several effects simultaneously allows one to...
engineer their relative strengths, improving the dynamic range and lowering the required signal intensity.

For a quasi-continuous signal with wavelength below silicon band edge, TPA and the induced free-carrier absorption (FCA) are the main sources of nonlinear loss in silicon waveguides. The evolution of optical intensity along the waveguide is described as [13]

\[ \frac{dI}{dz} = -\alpha I - \beta_{\text{TPA}} I^2 - \sigma \Delta N I, \]  

where \( \alpha \) is the linear loss coefficient; \( \beta_{\text{TPA}} = 5 \times 10^{-12} \text{ m/W} \) is the TPA coefficient, which is proportional to the imaginary part of third-order susceptibility; and \( \sigma = 1.45 \times 10^{-21} \text{ m}^2 \) is the cross section of free-carrier absorption, at 1550 nm [13]. At steady state, the free-carrier density \( \Delta N \) is represented by

\[ \Delta N = \frac{\beta_{\text{TPA}}}{2h\nu_0} I^2, \]  

where \( \tau \) is the free-carrier lifetime, and \( h\nu_0 \) is the photon energy.

The optical limiting phenomenon is observed at high input intensity as a result of the dominant nonlinear loss [14]. Between the linear region and the saturation region, there exists a sublinear curve that resembles a logarithmic function, as illustrated in Fig. 2. The logarithmic region is defined as the largest input intensity range whose output can be fit to a logarithmic function.

As a measurement for the fitting accuracy, two deviation calculation methods are employed. To evaluate the average accuracy of the computing primitive, the normalized root-mean-square error (NRMSE) should be no larger than 1% and is defined as

\[ \text{NRMSE} = \sqrt{\frac{(I_{\text{out}} - I_{\text{fit}})^2}{I_{\text{max}} - I_{\text{min}}}}. \]  

To ensure the accuracy of each single input value, the maximum error should be no larger than 3.5% and is defined as

\[ \text{Max Error} = \max \left( \frac{I_{\text{out}} - I_{\text{fit}}}{I_{\text{out}}} \right). \]  

An example of a waveguide with length \( Z = 2 \text{ cm} \), lifetime \( \tau = 1 \text{ ns} \), and propagation loss \( \alpha = 3 \text{ dB/cm} \) is shown in Fig. 2. The signal undergoes degenerate TPA, and FCA and is fit to a logarithmic function \( I_{\text{fit}} \) over the input intensity range 50 to 250 MW/cm², resulting in a 7 dB dynamic range. It is noted that it requires very high input power to reach the logarithmic region. This results from the large ratio between the linear loss coefficient and the nonlinear loss coefficient: the nonlinear term only comes into effect when input intensity is above a certain region. A low propagation loss coefficient and a large free-carrier lifetime would reduce this ratio and shift the logarithmic region to lower input intensity. Unfortunately, a large free-carrier lifetime is not practical because it also reduces the device’s speed, while ultra-low linear absorption is limited by the fabrication technology. A practical computing primitive thus would require larger logarithmic range, lower power, and more flexibility.

Stimulated Raman scattering offers optical gain in silicon without requiring phase matching [15]. The saturation of Raman amplification provides the opportunity to reach the logarithmic region with low input signal power. It also increases the dynamic range without significantly increasing setup complexity.

The Raman amplification in silicon, along with nonlinear absorption, can be modeled as [9]

\[
\begin{align*}
\frac{dI}{dz} &= (-\alpha - g_{R}\lambda_{0} I) I - \beta_{\text{TPA}} (I_{S} + 2I_{R}) I - \sigma \Delta N I, \\
\frac{dI_{S}}{dz} &= (-\alpha - \frac{g_{R}}{2\pi} I_{S}^{2}) I_{S} - \beta_{\text{TPA}} (I_{S} + 2I_{R}) I_{S} - \sigma \Delta N I, \\
\Delta N &= \frac{\beta_{\text{TPA}}}{2 \lambda_{0} h} (I_{S}^{2} + I_{R}^{2} + 2I_{R} I_{S}).
\end{align*}
\]

where \( g_{R} = 76 \text{ cm/GW} \) is Raman gain coefficient [8], and \( I_{R} \) is the Raman pump intensity. Without loss of generality, the wavelength dependence of the linear loss coefficient \( \alpha \), TPA coefficient \( \beta_{\text{TPA}} \), and FCA coefficient \( \sigma \) are ignored.

At low input signal intensity, the Raman pump source amplifies the output signal. The gain becomes less significant when the input signal grows, as the pump source is depleted by nonlinear absorption and amplification. Gain saturation modifies the input–output curve and expands the logarithmic region. Although it has a similar system setup, the logarithmic computing primitive functions fundamentally differently from a silicon Raman amplifier [9,11]. In the latter case, the signal intensity is significantly smaller than the pump. Under the assumption of negligible pump depletion, the output signal increases linearly with the input. In the logarithmic computing primitive case, both pump depletion and nonlinear absorption modify the output signal to be a sublinear function of the input. In addition, unlike the case in [11] where picosecond pulse signals are considered, the proposed signal focused on quasi-steady signal, where self-phase modulation and cross-phase modulation have minimum effect.

As shown in Fig. 3, a 10.5 dB logarithmic region for signal input from 0.035 to 0.4 MW/cm² is achieved when the input Raman pump is 91 MW/cm². The introduction of the amplification significantly reduces the power requirement on the signal power and increases the logarithmic range.

A numerical sweep of the input pump intensity shows that at 48.5 MW/cm², the input logarithmic range is further expanded to 17.5 dB, from 0.4 to 22.4 MW/cm², as shown in Fig. 4. The Raman pump expands device flexibility, allowing one to trade between Raman pump intensity, signal intensity, and logarithmic range. We also note that scaling the signal intensity before or after the logarithmic step allows one to increase or decrease the range of valid signal intensities (though we acknowledge that the dynamic range remains at best unchanged).
Although the use of a Raman pump can immensely reduce the required signal intensity (see Figs. 3 and 4), the output deviates from a logarithm at higher signal intensities. To shape the curve at high input intensity, a new pump source $I_p$ is injected into the waveguide to enhance the nonlinear absorption process through nondegenerate TPA with the signal wave. The evolution of signal wave $I_s$, Raman pump wave $I_R$, and nondegenerate TPA pump wave $I_p$ can be modeled as

$$\frac{dI_s}{dz} = \left(-\alpha + g_R I_s\right) I_s - \beta_{\text{TPA}} (I_s + 2 I_R + 2 I_p) I_s - \sigma \Delta N I_s$$

$$\frac{dI_R}{dz} = \left(-\alpha - \frac{\alpha}{2} g_R I_s\right) I_R - \beta_{\text{TPA}} (I_R + 2 I_s + 2 I_p) I_R - \sigma \Delta N I_R$$

$$\frac{dI_p}{dz} = -\alpha I_p - \beta_{\text{TPA}} (I_p + 2 I_s + 2 I_R) I_p - \sigma \Delta N I_p$$

$$\Delta N = \frac{\sigma \beta_{\text{TPA}}}{2 \Delta \nu_0} (I_s^2 + I_R^2 + I_p^2 + 2 I_s I_R + 2 I_s I_p + 2 I_R I_p)$$

(6)

The nondegenerate TPA with the third beam $I_p$ suppresses the output signal as the input increases, extending the logarithmic range at the high input side. For an input Raman pump $I_R$ at 56.1 MW/cm$^2$ and the input nondegenerate TPA pump source $I_p$ at 50.1 MW/cm$^2$, the logarithmic input range is enlarged to 19.5 dB, from 0.32 to 28.2 MW/cm$^2$, as shown in Fig. 5. Note that the optimized initial pump intensity varies with wavelength due to the nonlinear coefficient's dependence.

One important application of the silicon photonic logarithmic device is for recovery of a signal of interest in the presence of multiplicative distortion [16]. This technique exploits the fact that the logarithm of the product of two inputs is the sum of the logarithms of those inputs. This allows the filtering of multiplicative noise by logarithmic filtering and conventional linear time-invariant filtering. Figure 6 illustrates this application. As explained in the caption, the logarithmic device followed by a linear filter can de-convolve and recover the signal from a mixed composite.

Compared to the synthesis of the logarithmic computing primitive, the scaling and the natural exponentiation computing primitive are implemented in a more straightforward way.

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**Fig. 3.** Similar to Fig. 2, a simulation of the logarithmic computing primitive wherein Raman amplification, along with concomitant nondegenerate TPA, is added to increase the dynamic range and vastly reduce required signal intensity. The input Raman pump intensity is 91 MW/cm$^2$. The output is fit to a logarithmic function over a 10.5 dB input range with a normalized-mean-square error of 0.99% and the maximum error of 3.2%.

**Fig. 4.** Simulation of the logarithmic computing primitive under the same conditions as in Fig. 3 (the signal undergoes saturated Raman amplification and nondegenerate TPA), except that the input Raman pump intensity is lowered to 48.5 MW/cm$^2$. Reduced Raman pump requirements and an increased logarithmic dynamic range of 17.5 dB are gained at the expense of higher required signal intensity. The normalized-mean-square error is 0.81% and the maximum error is 3.4%.

**Fig. 5.** Synthesis of the logarithmic computing primitive with the nonlinear-absorption-enhanced Raman amplification. The input Raman pump is 56.1 MW/cm$^2$, and the input nondegenerate TPA pump source is 50.1 MW/cm$^2$. The output is fit to a logarithmic function over a 19.5 dB input range with a 0.99% normalized-mean-square error of 0.99% and a 3.1% maximum error.

**Fig. 6.** The silicon photonic logarithmic device can perform signal de-convolution. It can be used to recover a signal of interest when it has been mixed (multiplied) by an unwanted signal of different frequencies. The figure shows the composite signal (dashed red) consisting of a single tone input mixed with two unwanted higher frequency tones. Linear filtering (dashed dot black) is unable to recover the input. Logarithm followed by linear filter (and natural exponentiation) is able to recover the input (solid blue). In both cases, the linear filter is a 10th order Butterworth.
One way to perform the scaling function is to use a variable optical attenuator. A p-i-n diode structure fabricated on an Si waveguide attenuates the input optical beams as a function of the current injection density [17,18].

Another way to implement the scaling primitive is to use the third-order parametric process. The output wave at a new frequency wave \( \omega_4 \) is generated from the mixing of the input waves at \( \omega_1, \omega_2, \) and \( \omega_3 \) as [19]

\[
\frac{dE_4}{dz} = -\frac{\alpha}{2}E_4 + \frac{2in_2\omega_4}{c}E_1E_2E_3^*e^{-i\Delta k_z}, \tag{7a}
\]

\[
\rightarrow \frac{dI_4}{dz} = -\alpha I_4 + \frac{2n_2\omega_4}{c^2\varepsilon_0 n} \sqrt{I_1I_2I_3} \sin \phi, \tag{7b}
\]

where \( E \) is the electric field amplitude, \( \Delta k \) is the phase mismatch, \( \sin \phi = 1 \) for perfect phase match, \( n \) is the refractive index, and \( n_2 \) is the nonlinear-index coefficient, which is proportional to the real part of third-order susceptibility. Under the low depletion assumption and perfect phase matching, Eq. (7b) calculates the output intensity of \( I_4 \) at distance \( l \), and the result scales with the input signal \( I_1(0), I_2(0), I_3(0) \):

\[
I_4(l) = \frac{4n^2_2\omega^2_4}{c^2\varepsilon_0 n^2} \left( 1 - \exp \left( -\frac{\omega^2_4}{2} \right) \right)^2 I_1(0)I_2(0)I_3(0). \tag{7c}
\]

The natural exponentiation primitive can be realized with the Raman amplification process. When the signal is significantly smaller than the pump source and the nonlinear absorption is negligible, the output signal is solved as [20]

\[
I_4(l) = I_4(0) \exp(-\alpha l) \exp(g_R L_{\text{eff}} I_0(0)), \tag{8}
\]

where \( l \) is the length of the amplifier and \( L_{\text{eff}} = (1 - \exp(-\alpha l))/\alpha \) is the effective length.

Exploiting the nonlinear optical properties native to silicon, we show an approach to create a logarithmic analog co-processor in silicon photonics. By engineering the relative strength of Raman amplification and nonlinear absorption, the sublinear relationship between signal input and output is tuned to emulate a logarithmic function. The logarithmic computing primitive, together with a scaling primitive and a natural exponentiation primitive, can be used sequentially to realize the extremely nontrivial analog optical exponentiation operation.

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